Testing and Analysis

Measuring the Dynamic Properties of Elastomers for Analysis

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Abstract
The analysis of small amplitude vibrations in deformed viscoelastic solids (Morman and Nagtegaal (1983), Morman, Kao, Nagtegaal (1981) [4,5] is applicable to rubber support mounts, automotive door seals [3] and vibration isolation gaskets. Use of these capabilities in analysis software such as ABAQUS or MSC.MARC requires the equilibrium large strain material properties and the dynamic material properties. An outline for the experimental determination of these properties at dynamic vibration frequencies between 0.01 Hz. and 20,000 Hz. is provided.

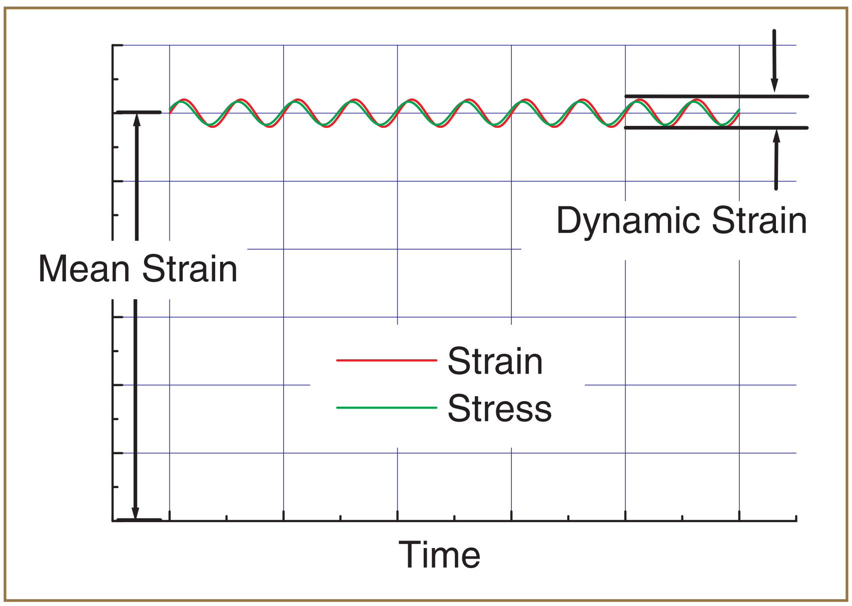

Figure 8, Small Amplitude Vibration on Large Mean Strain

Introduction
The analysis of small amplitude vibrations in deformed viscoelastic solids requires knowledge of the equilibrium large strain material properties and the dynamic material properties. The experiments used to determine these material properties may be different than those commonly performed in elastomer laboratories. Most of the experimental techniques described herein to meet these needs are presented in the referenced literature but may not be implemented in commercially available test instruments

In practice, all aspects of the material properties of engineering elastomers cannot be represented by one analytical model. Some judgement is required to determine which material properties are the most significant. In the analysis of dynamic properties, it is important to understand the overall material sensitivity to mean strain level, dynamic amplitude and vibration frequency so that a subset of these properties may be judiciously selected and used in analysis.

In general the experimental approach is as follows:

A. Establish large strain non-linear equilibrium material properties in the range of interest.
B. Determine the dynamic properties of superimposed linear vibrations in the frequency, the mean strain and the dynamic amplitude ranges of interest.

Developing Equilibrium Stress Strain Curves

If an elastomer is stretched to a particular strain and held, the stress in the elastomer will decrease over time. This decrease in stress over time is referred to as stress relaxation. This reduction in stress can be a significant fraction of the initial stress.

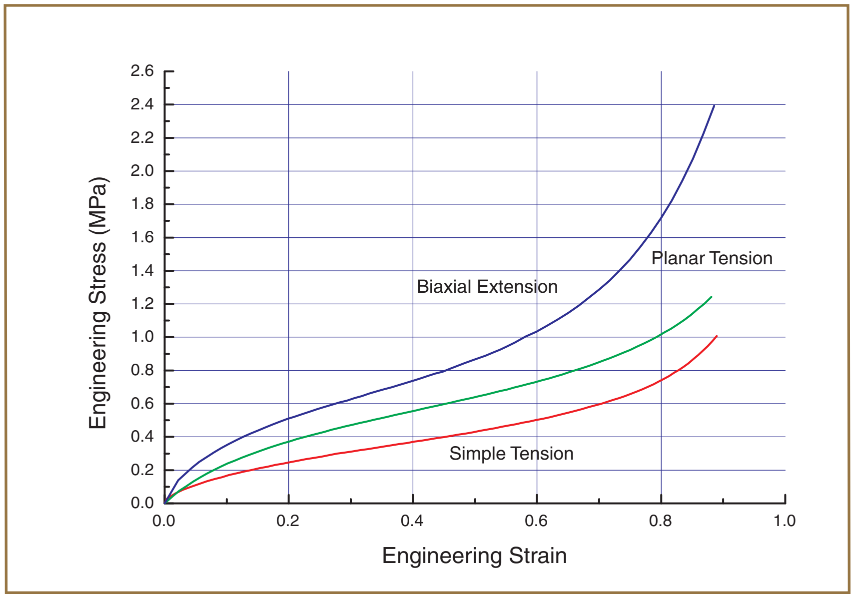

Figure 2, Typical Data for 3 Basic Strain Sates

To capture the equilibrium condition of the elastomer, the material must be allowed to relax and "relaxed" stress-strain curves need to be developed. One way to accomplish this is to allow the material to relax at several different strain levels. At each strain level, record the equilibrium stress value. Naturally, some judgement is required to determine a relaxation time interval. The resulting equilibrium stress-strain pairs are then connected and used to create an equilibrium stress-strain curve (Figure 6).

This same loading sequence would be used in simple tension, planar tension and biaxial extension. This set of curves is then used to fit a hyperelastic material model.

Dynamic Properties

The dynamic material properties of interest are the storage modulus and loss modulus of the material when subjected to a sinusoidal vibration superimposed on a mean strain. The storage modulus and the loss modulus are the real and imaginary parts of the dynamic modulus [2]. In this case, for input into the analysis software, we are interested in the differential dynamic modulus described by Nolle [6] as follows:

"Differential dynamic modulus refers to dynamic modulus determined from a sample upon which a static deformation is imposed along the same coordinate as the dynamic deformation. Because the differential dynamic modulus is a complicated function of static strain for many rubbers, it is best to regard the sample at any particular static deformation as a distinct material from the standpoint of dynamic modulus measurements. … The differential dynamic modulus cannot be identified with the slope of the quasi-static stress-strain curve except at very low frequency."

In this discussion, dynamic modulus will be measured as differential dynamic modulus. The dynamic modulus measurements are not simply a function of frequency and therefore need to be made at mean strain levels appropriate to the application. Other practical implications are noted in the following sections.

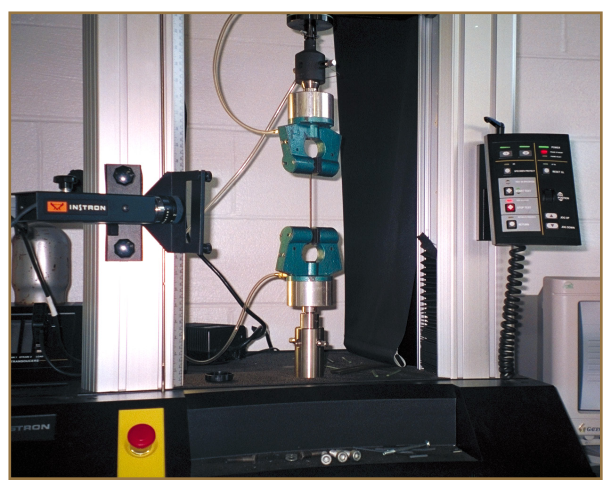

Figure 3, Simple Tension Experiment

Measuring Dynamic Properties at Frequencies from 0.01 Hz. to 400 Hz.

In the frequency range of 0.01 Hz. to 400 Hz. the mass of the specimen may be ignored and the wavelength of the vibration is assumed to be very large compared to the length of the specimen (Figure 7). Experiments can be performed in simple tension, planar tension or both.

A Instron Model 8500 [8] servo-hydraulic test instrument is used to transmit sinusoidal vibrations into a statically deformed specimen (Figure 8). It is critical that the material first be at equilibrium. If the material is not at equilibrium, the stress relaxation of the material will alter the dynamic vibration measurements. One approach is to perform the dynamic properties measurements on the same specimen and during the same experiment used to determine the large strain equilibrium properties. After the specimen has reached static equilibrium at a particular mean strain level, one can introduce a series of vibrations.

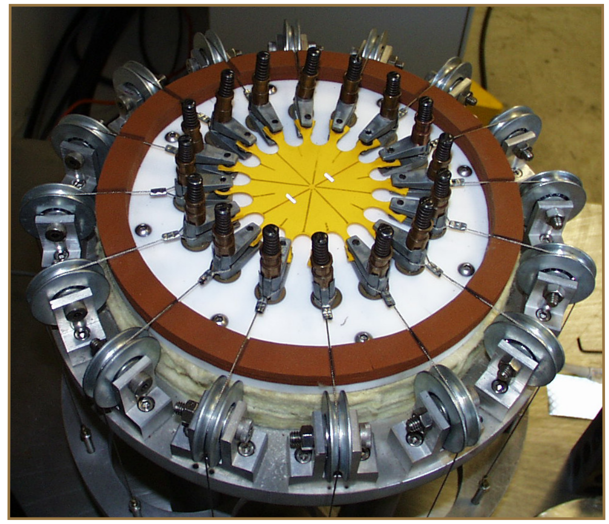

Figure 4, Equal Biaxial Experiment

The experimental procedure is to introduce a sine wave vibration into the specimen. The actual strain amplitude, stress amplitude and the phase angle (δ) between the two sine waves is then measured (Figure 9). From these basic measurements, the dynamic modulus and associated moduli can be calculated as follows:

Dynamic Modulus (E*) = (Stress Amplitude)/ (Strain Amplitude)
Storage Modulus = E* cosδ
Loss Modulus = E* sinδ

The vibration needs to be of sufficient duration to make an acceptable measurement without causing the specimen to significantly change temperature. To obtain the differential dynamic modulus, the dynamic strain amplitude is based on the size of the deformed specimen and the dynamic stress measurement is based in the smaller deformed specimen cross sectional area. This is different from the static equilibrium data that is typically reported as engineering strain and engineering stress by convention and input into hyperelastic curve fitters in that form.

The dynamic properties of elastomers are sensitive to the mean strain upon which they are superimposed (Figure 10). The mean strain influence can be more significant in certain applications than the frequency influence.

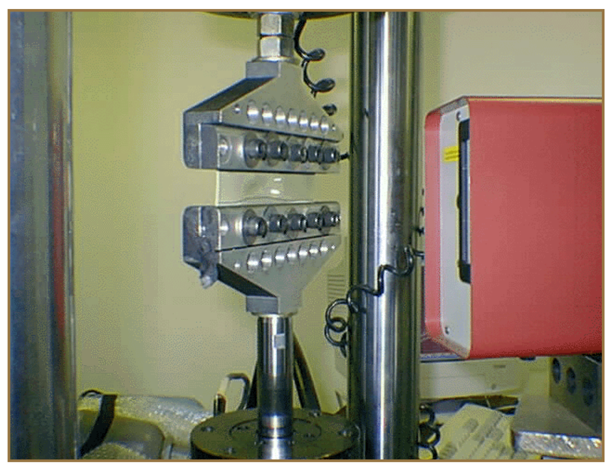

Figure 5, Pure Shear Experiment

For unfilled elastomers, the dynamic modulus is insensitive to the vibration amplitude. However, most engineering elastomers are filled materials and can be extremely sensitive to vibration amplitude unless the vibration amplitudes are very small. Most vibrations associated with rubber support mounts for example would be of sufficient magnitude to justify concern regarding vibration amplitude. Increases in filler content generally cause increases in vibration amplitude sensitivity [7](Figure 11). Because of this, in addition to testing across the frequency range of interest and the mean strain range of interest, it is imperative to test across the vibration amplitude range of interest.

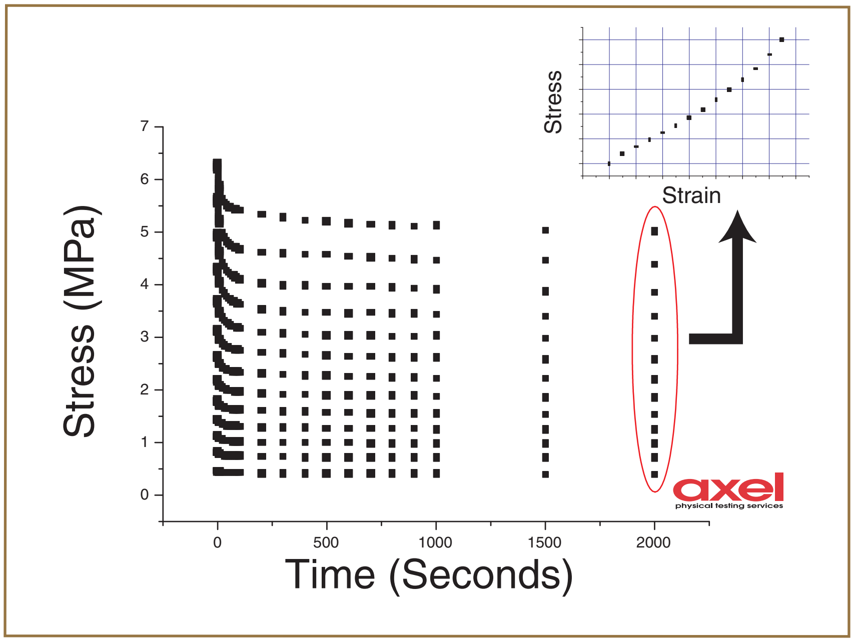

Figure 6, Building an Equilibrium Stress-strain Curve from Stress Relaxation Data

The dynamic testing results in a large matrix of storage modulus and loss modulus values at various frequencies, amplitudes and mean levels. Given the range of values and the design concerns of the application, the analyst must decide which conditions to actually simulate.

Measuring Dynamic Properties at Frequencies from 400 Hz. to 20,000 Hz.

As vibration frequencies exceed 400 Hz., the dynamic vibration wavelength becomes too small to be considered very large compared to the specimen length. As such, one needs to consider the effect of the wave travelling through the specimen. The mass of the specimen becomes significant (Figure 12). A method to measure the dynamic properties in the range of 400 Hz. to 20,000 Hz. is to perform wave propagation experiments [6]. In these experiments, the rate of travel and the attenuation of the wave amplitude of a travelling wave in a long thin strip of elastomer are measured.

The experimental procedure is to stretch a long strip of elastomer to the mean strain level of interest and allow it to reach an equilibrium condition (Figure 13). The specimen needs to be very narrow such that the width and thickness are very small relative to the dynamic vibration wavelength yet very long such that a high frequency travelling wave doesn't reach the end and reflect back into the

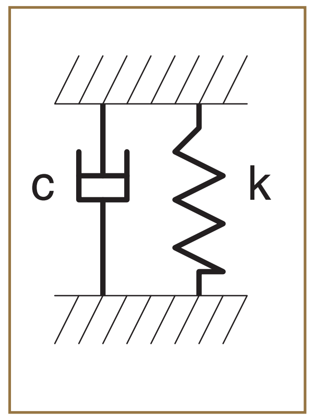

Figure 7, Low Frequency Specimen Model

Figure 8, Tension Specimen in Servohydraulic Test Instrument

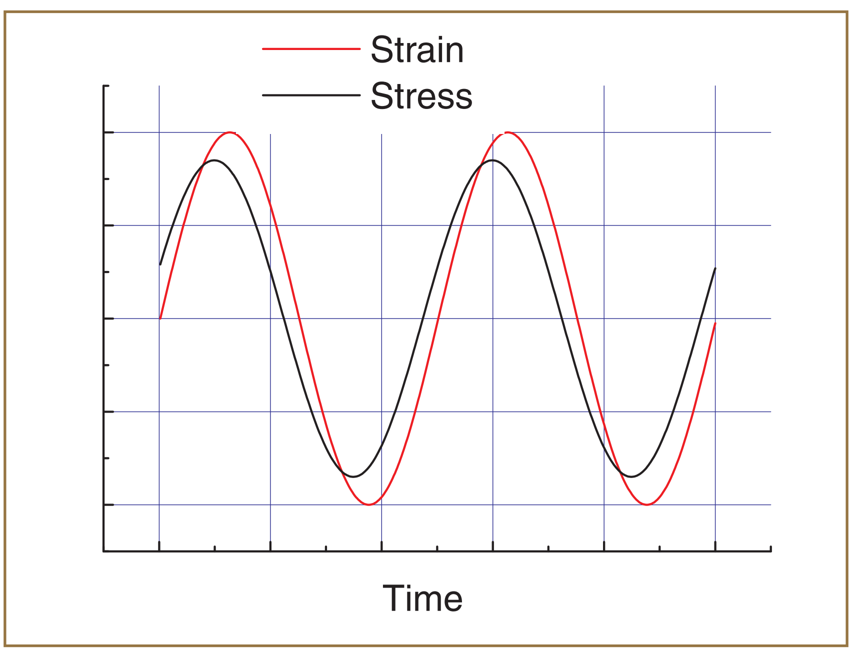

Figure 9, Experimental Sinusoidal Input with Sinusoidal Response measurement. A small sinusoidal dynamic vibration of short duration is introduced at one end of the long specimen so as to produce a longitudinal travelling wave. An accelerometer is used to measure the vibration at the source and a position sensor is used to measure the amplitude of the wave as it travels past a point which is a known distance along the specimen. From this a phase relationship between the source and the point and an amplitude relationship between the source and the point is determined. The same dynamic vibration is again introduced and the phase relationship and amplitude relationship to a different point is determined.

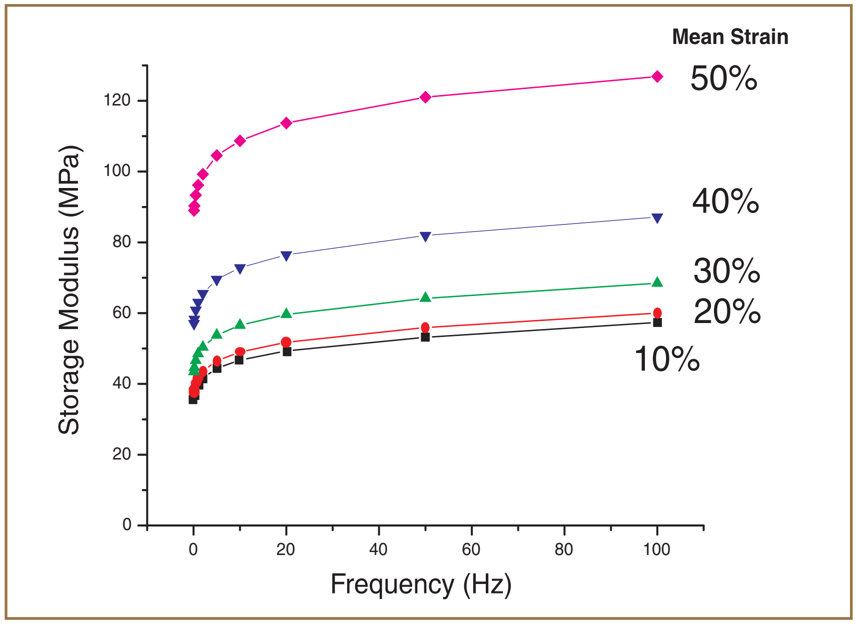

Figure 10, Typical Mean Strain Effect on Storage Modulus

Given relative measurements at two known points along the specimen, the phase and amplitude relationship between the two points becomes known. From the phase of the wave between points a known distance apart, the wavelength λ can be calculated. The speed *c* of the longitudinal wave is given by:

$$c = f\lambda$$

where f is the vibration frequency. If the density (ρ) of the material is known, from basic wave equations the dynamic modulus can be calculated as follows:

Dynamic Modulus $E^* = \rho c^2$

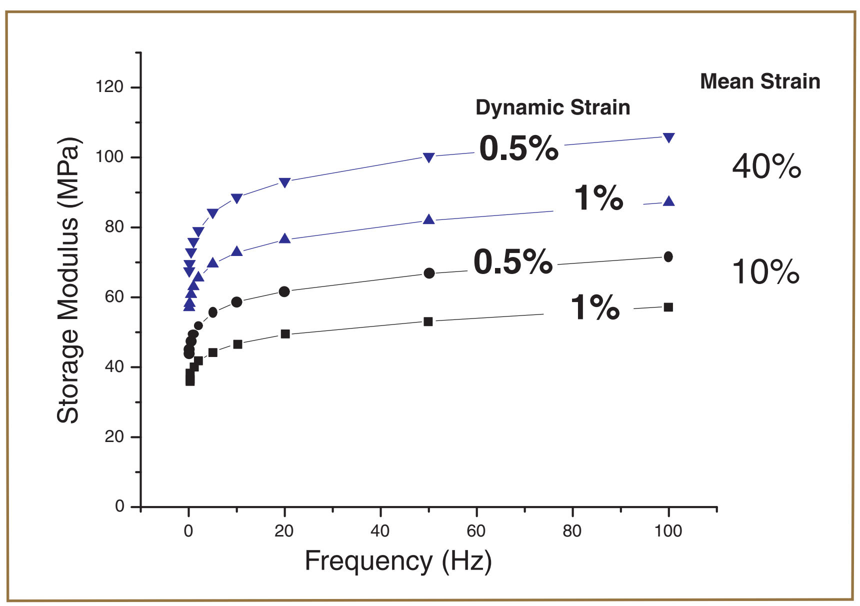

Figure 11, Typical Dynamic Amplitude Effect on Storage Modulus

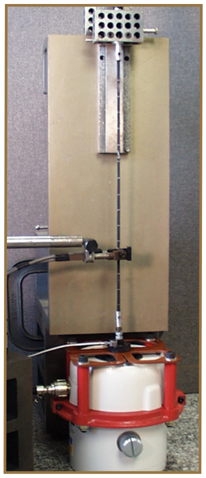

Figure 13, Wave Propagation Experiment

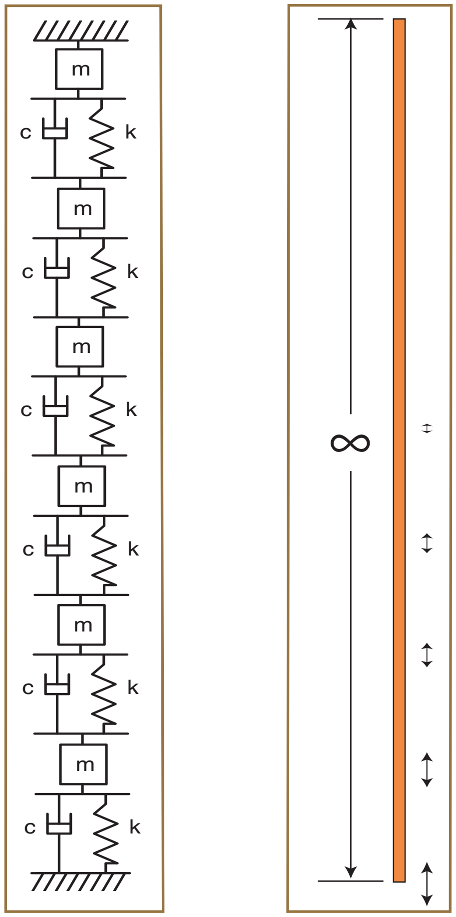

Figure 12, High Frequency Material Representation including Mass Effects

The storage modulus and the loss modulus corresponding to the dynamic modulus may be determined using the amplitude attenuation measurements [6]. The advantage of the wave propagation experiment is that high frequency dynamic properties can be measured directly. The disadvantage is that the amplitudes for this type of experimental apparatus tend to be very small. Because the amplitudes are very small, they are relatively insensitive to the absolute value of dynamic strain. This may not be critical because high frequency vibrations in engineering application are often very small.

Summary

The dynamic properties of elastomers can be directly measured across a broad range of frequencies and made available for analysis (Figure 11). For the case of small amplitude vibrations in deformed viscoelastic solids, the large strain non-linear equilibrium material properties must first be measured followed by superimposed dynamic vibrations using the appropriate experiment. Because the mean strain and the dynamic amplitude influence can be sigificant, some judegement must be expercised in the selection of experimental conditions and simulation conditions.

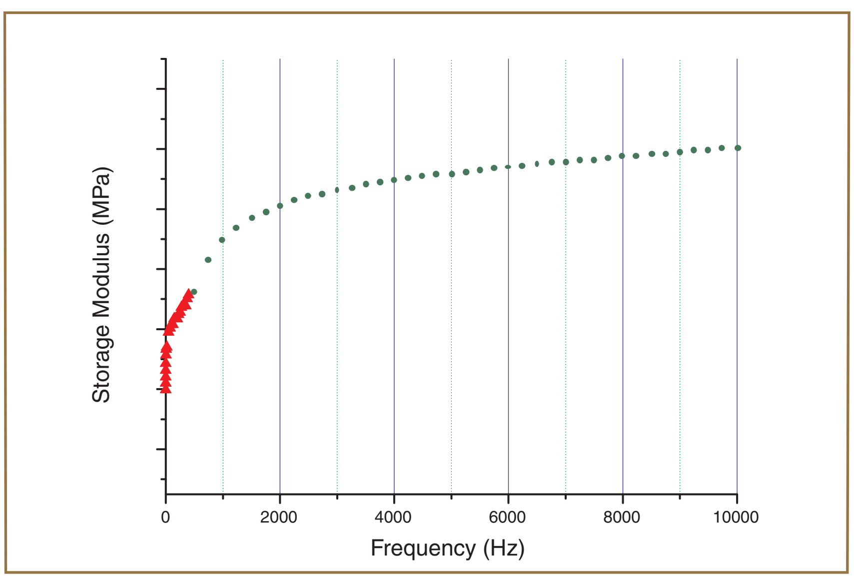

Figure 14, Storage Modulus Measurements for 1 Material using both Low Frequency and High Frequency techniques to generate data over a wide Frequency Range

References

1. J. R. Day, and K.A. Miller, "Equibiaxial Stretching of Elastomeric Sheets, An Analytical Verification of Experimental Technique" ABAQUS 2000 User's Conference Proceedings, Newport, Rhode Island, May 30-June 2, 2000

2. J D. Ferry, Viscoelastic Properties of Polymers, 3rth ed. (Wiley, New York, 1980)

3. Y Gur and K N Morman, Jr., Analytical Prediction of Sound Transmission Through Automotive Door Seal Systems, Presented at The Third Joint Meeting of the Acoustical Society of America and the Acoustical Society of Japan, Honolulu, Hawaii, December 2-6, 1996

4. K N Morman, Jr., and J C Nagtegaal, Finite Element Analysis of Sinusoidal Small-Amplitude Vibrations in Deformed Viscoelastic Solids. Part I. Theoretical Development, International Journal for Numerical Methods in Engineering, Vol. 19, pp.1079-1103 (1983)

5. K N Morman, Jr., B.G. Kao and J C Nagtegaal, " Finite Element Analysis of Viscoelastic Elastomeric Structures Vibrating about Non-Linear Statically Stressed Configurations", SAE Paper 811309 (1981)

6. A W. Nolle, "Methods for Measuring Dynamic Mechanical Properties of Rubber-Like Materials," J. Appl. Phys. 19, 753 (1948)

7. A.R. Payne, and G. Kraus, Reinforcement of Elastomers (Interscience Publishers, New York, 1965)

8. Instron Corporation, Canton, Massachusetts

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Axel Products provides physical testing services for engineers and analysts. The focus is on the characterization of nonlinear materials such as elastomers and plastics.

Data from the Axel laboratory is often used to develop material models in finite element analysis codes such as ABAQUS, MSC.Marc, ANSYS, Endurica and LS-Dyna.

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